IMPERIAL BUSINESS SCHOOL

Pricing Discrete and Nonlinear Markets With Semidefinite Relaxations

Cheng Guo†, Lauren Henderson†, **Ryan Cory-Wright***, Boshi Yang†

Affiliations: † Clemson University, SC, USA | * Imperial Business School, London, UK | Website: ryancorywright.github.io

QR code for paper

Abstract: We use sensitivity analysis to price non-convex markets via their semidefinite relaxations, obtaining smaller lost opportunity costs than via existing LP-based relaxations. **Keywords:** Electricity markets, non-convex pricing, semidefinite programming.

Why Pricing Non-Convexities Is Hard

Well-designed electricity markets have three properties:

1. **Efficient dispatch**: service demand at minimum cost
2. **Individual Rationality**: pay generators enough so efficient dispatch maximizes their profit
3. **Revenue adequacy:** collect enough from demand to fund market payments

Strong duality delivers all three properties in convex markets

Real-world markets have non-convex UC and AC load-flow constraints that break dual-based pricing and require uplift payments (subsidies) ranging into the $100 millions per year

Research question: how to obtain tractable marginal prices from a non-convex market clearing model, thereby reducing subsidies paid to generators by consumers.

Proposal: Relaxation-Based Pricing

- **Clear the market:** solve non-convex resource allocation problem for a feasible or optimal dispatch
- **Price the market:** replace non-convex problem by its SDP/RLT relaxation, set prices according to sensitivity analysis of the relaxation value function
- **Justification:** Dual prices to compute marginal value of demand justified via the envelope theorem (Milgrom and Segal 2002, Theorem 1)

Envelope theorem [2, Thm. 1]: At differentiable points, right-hand side sensitivities are given by optimal dual variables. Moreover, sensitivities are unique wherever the SDP value function is differentiable, i.e., almost everywhere.

Why Tighter Relaxations Help

- Lost opportunity cost (LOC) is the gap between a participant's best possible profit at posted prices and realized profit under the feasible allocation
- A higher LOC means a worse bound on out-of-market charges to consumers in theory, and more out-of-market charges to consumers in practice. Formally:

Total LOC bound [2, Thm. 2]: Let the dual prices be subgradients of the relaxation value function. Then:

$$\text{Total LOC} \leq \text{Relaxation Gap}.$$

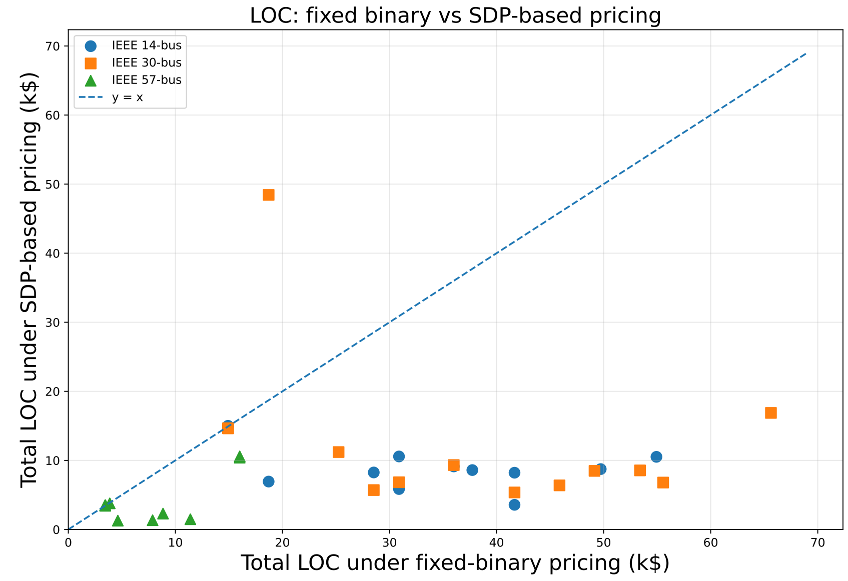
- Tighter SDP relaxation means smaller optimality gap
- Smaller gap implies fewer subsidies are required to align participants' incentives

Numerical Results

- Both DCUC and ACUC semidefinite relaxations are much tighter than linear relaxation counterparts
- Comparison with fixed-binary pricing on 39 DCUC instances:

46% avg. LOC reduction	**31/39** lower-LOC cases
2.1% avg. SDP gap	**5.4%** avg. LP gap

Total LOC Comparison



Points below the diagonal indicate lower LOC under SDP pricing.

Summary

- Using convex relaxations, we develop new pricing mechanisms for DCUC and ACUC markets that send **fair and transparent** price signals
- LOC bound formalizes intuition that tighter relaxations give better operational guarantees
- Framework generalizes to other mixed-integer quadratic resource allocation problems, e.g., allocating landing slots at airports, fisheries markets

References

1. Guo, Bodur, and Taylor (2026). Copositive duality for discrete energy markets. Management Science 72(3):2022–2040.
2. Guo, Henderson, Cory-Wright, and Yang (2026). Pricing discrete and nonlinear markets with semidefinite relaxations. arXiv 2602.15722.
3. Milgrom and Segal (2002). Envelope theorems for arbitrary choice sets. Econometrica 70(2):583–601.